Knot News

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Aspects of Sphere Covering Knots

by Pieter van de Griend

If a knot or other covering is to be tied regularly with one cord, it is limited to a one-cycle diagram.

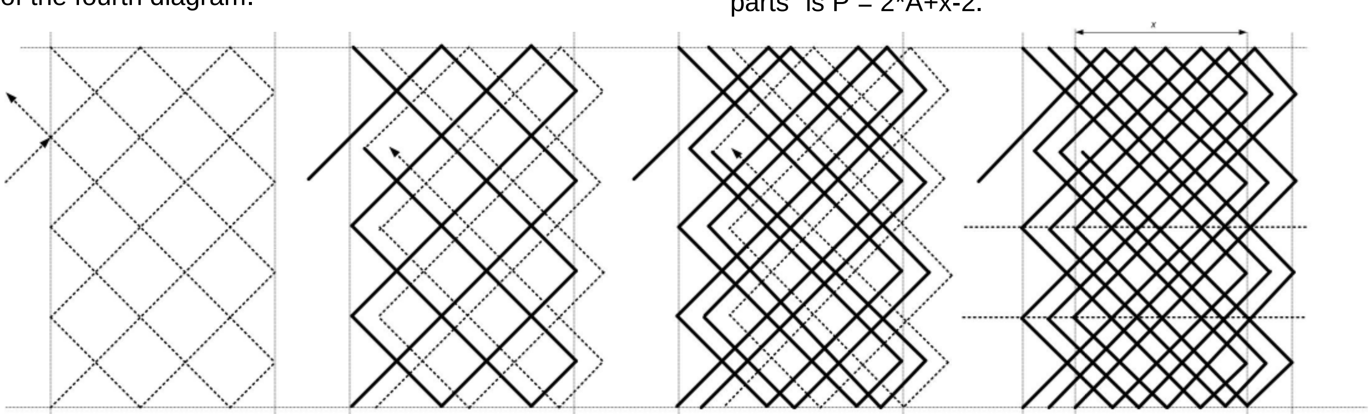
Clifford Warren Ashley [2, p357, #2216].

Prologue

Ever persisted in doubling a Turk's Head Knot till you got fed up? During this multiplying process the working end keeps on following the standing end on the same side. Did you take notice of what actually happens? In Fig.1 you can see what curious process takes place if you'd do this for a 5 part 4 bighted Regular Knot Grid [6]. The first diagram shows the starting configuration. The second diagram starts multiplying below the stend. Note that this type of operation is not by any means comparable to the usual expansion of a Regular Knot Grid. In fact the whole point is that we are obtaining a totally *different grid-type* here altogether. This can be checked by considering the length block between the dotted lines of the fourth diagram.

Nested Knots

We just saw the Regular Knot grid-type transform into a new species of grid during a simple operation. In the literature this grid-type is known as a Nested Knot Grid [7]. This name is easily explained. There are a number of "nests of bights" along either rim. This already gives two parameters to classify this grid-type, namely the number of nests (*B*) and the depth of the nests (*A*). In our example we happened to have chosen a 5 part 4 bighted Regular Knot Grid to start from. This multiplies into a wider knot than, should we have started from, say, a 3 part 4 bighted Regular Knot. So, there must also be some parameter to denote the width of these Nested Knot grids' weave. I call this segment the Equatorial Weave, for reasons which will become clear later. Its number of parts is usually called *x*. From Fig.1 we can also see that there is something like a vertical shift between the left and right-hand outermost rims. This is denoted by *y*. Sweeping these ideas together we find a 4-parameter notation for Nested Knots (*B*,*A*,*x*,*y*). The "length in bights" of Nested Knots is readily found by the product *A*B*. The "width in parts" is P = 2*A+x-2.



Multiplying a 5 part 4 bight Regular Grid

Fig.1

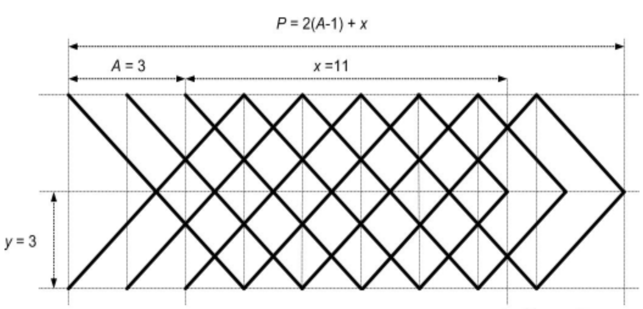
It is left as an exercise for the reader to verify that, when *A*=1, our good old Regular Knot Grids appear.

Another interesting facet is the behavior of the Law of the Common Divisor for Nested Knots. For Regular Knots it states that they will be single stranded provided *p*, the number of parts, and *b*, the number of bights, are co-prime. In other words, the greatest common divisor between parameters *p* and *b* equals one. For our Nested Knots the situation is more complicated. After all, we now have 4 instead of 2 parameters to deal with. However, for those who are interested in single stranded Nested Knots, try looking at co-prime combinations of (*B,P*) and (*A*,*y*).

Another truly bizarre feature, which Nested Knots may surprise you with, is that of negative *x*-values! I challenge you try and find one!

Covering Spheres

What is so special about Nested Knots? Well, if *x* is not too large and *B*=4, you have neat Sphere Covering grids. Actually Nested Knots for which *B*=4 and for which *x* is not some excessive positive value are also known as short-sausage-coverings (SSC) rather than Single-stranded Sphere Coverings!



Nested Knot Lengthblock (1,3,11,3) **Fig.2**

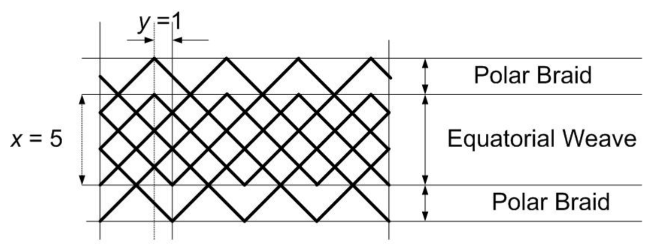
Bibliographical notes

If you set out to find it, there is some interesting literature on Sphere Covering Knots. This is quite understandable, as decorative knotting often requires sphere coverings. There are many ways into the subject. Some solutions, such as the Globe Knot are very makeshift [9], [10, p189, #496]. Grant and Lopez get their SSC by means of interweaves [3], [5, pp219-223]. Ashley was probably the first keen researcher [1], [2], who approached the subject in some systematic manner. With our new notation in hand we can translate some of the famous examples from *Ashley's Book of Knots*.

Sphere Covering Knot samples in Ashley	
(3,2,3,1)	#1391
(4,3,2,1)	#2216
(4,2,5,1)	#2217
(4,2,6,2)	#2218 after repair by Scott [8]
(4,2,10,2)	#2219 after repair

Some naming conventions

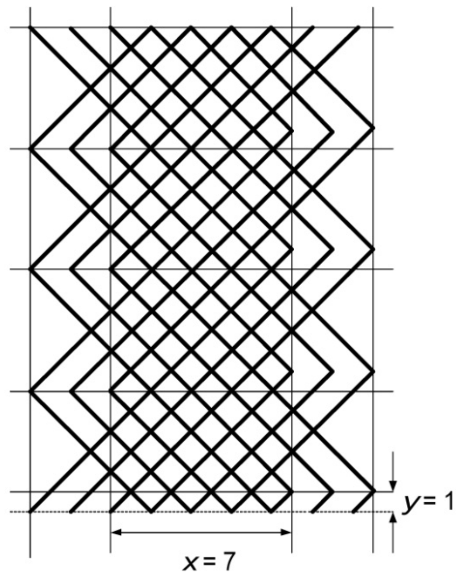
These knots can cover spheres. Therefore it might be easier to occasionally speak in terms of navigation rather than numbers and cryptic symbols. Consider (4,2,5,1). Put the structure on its side, like in Fig.3. One can discern three horizontal bands of weave, two Polar Caps-braids and a strip of Equatorial Weave. The band of Equatorial Weave has width *x*=5. The Polar Caps have a (smallest) rotational shift of *y*=1 relative to each other.



Nested Knot (4,2,5,1) **Fig.3**

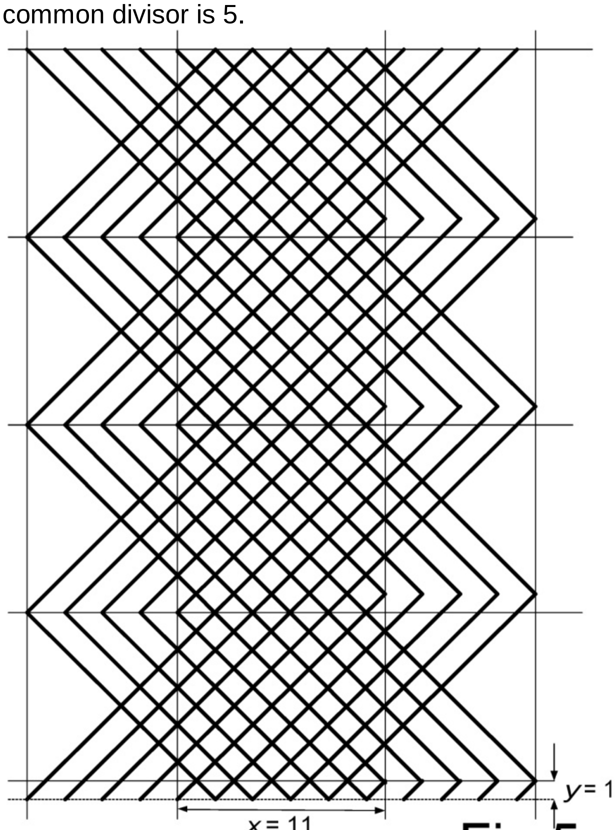
Symmetrical Nested Knots of 1 strand

Let us consider some examples of single stranded sphere coverings. As we just said *B* must be 4 in order to have the 4-valent opening at the top and bottom of the sphere. We say that the Polar Openess must be 4. Depending on how large *A* is chosen to be, we push the central section of regular weave, the so-called Equatorial Weave, away from the poles. But let's make *A* equal to 3. This allows us to connect a piece of Equatorial Weave of *B* times *A* bights (which is 12 bights) and x parts to our polar caps. If we want a single stranded SSC to occur then the rotational shift between the caps, *y*, must be co-prime with A. Some study will show that *y* is determined by the parity of *x*, among other things, but if *x* is odd, then *y* must be odd too. Let's assume we have an Equatorial Weave of *x*=7, then our *y*-value can be 1,3,5,7… Let *y*=1. As (*A*,*y*) = (3,1) is also a co-prime pair. Our Nested Knot will be single stranded. Check this out in Fig.4.



Nested Knot (4,3,7,1) Fig.4

A more ambitious Nested Knot Project is (4,5,11,1). It is shown in Fig.5 Note that (4,5,11,3), (4,5,11,7), (4,5,11,9) will also be single stranded, but (4,5,11,5) has 5 components! This is because (*A,y*) = (5,5) does not contain 2 co-prime numbers. In fact their greatest common divisor is 5.

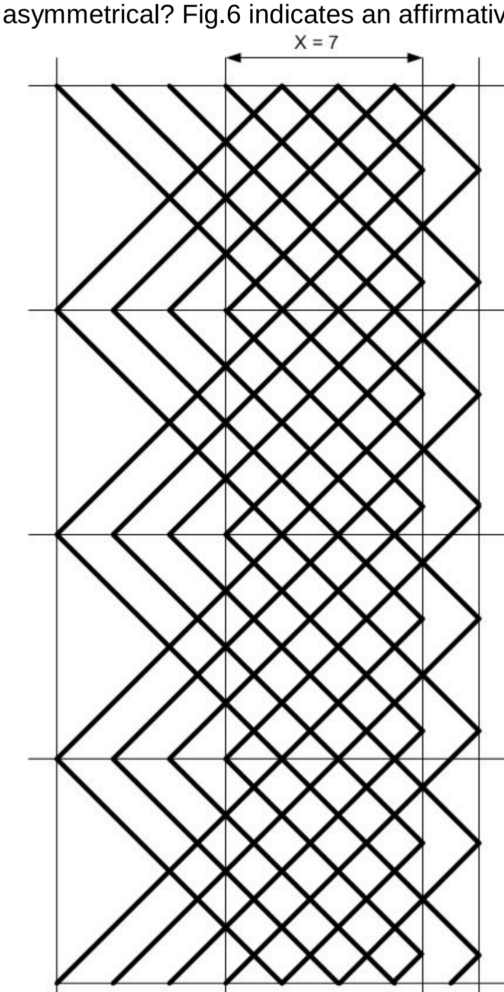


Nested Knot (4,5,11,1) Fig.5

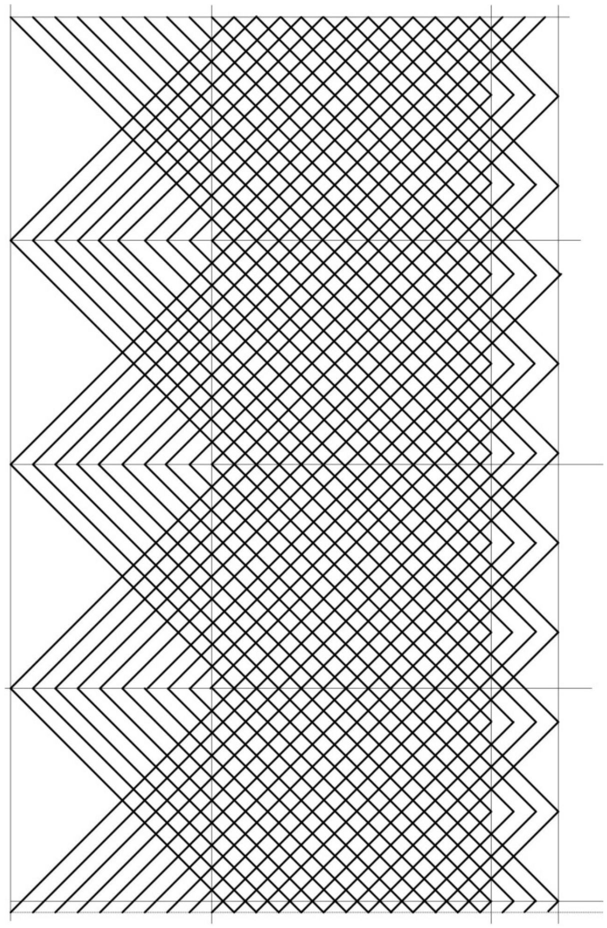
Punctured Spheres

The practical aspects of the sphere covering problem are not quite solved with a Symmetrical Nested Knot. The ball at the end of a bell pull can be covered with a covering like [1, #2216]. The problem with that kind of covering is that they are completely closed and leave no room for emerging parts, such as the stem or a tassel of the bell pull. This problem can be solved by cutting a hole in a sphere coverer. This leaves one with a sphere with a hole, hence the name “punctured sphere” [4].

So far we have considered Symmetrical Sphere Coverers which have identical Polar Caps. However, there is no need to hold onto this property. Key is the product of *A* and *B*. It determines the length of the Equatorial Weave and may well factor into some different values. In our first example we had *B*=4 and *A*=3. This yielded 4*3 =12. But 12 is also 6 times 2. Can we find single stranded structures which are asymmetrical? Fig.6 indicates an affirmative answer.

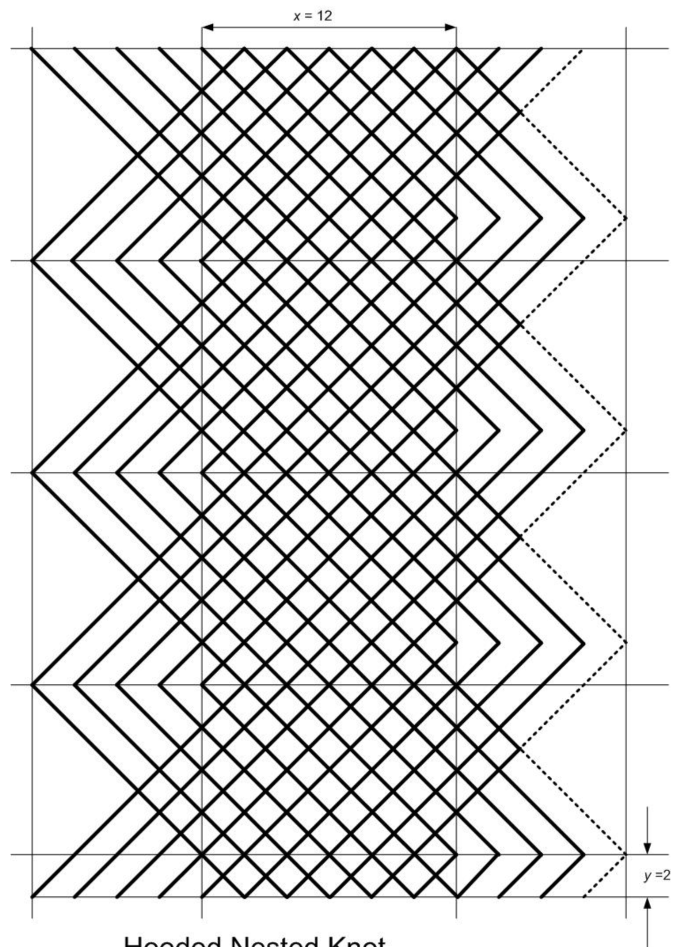


Asymmetric Nested Knot (4/4, 7,1, 8/2) Fig.6



Asymmetric Nested Knot
(4/10, 25,1, 10/4)

Fig.7



Hooded Nested Knot
(4,5,12,2) r-tim 1

Fig.8

Hooded Nested Knots

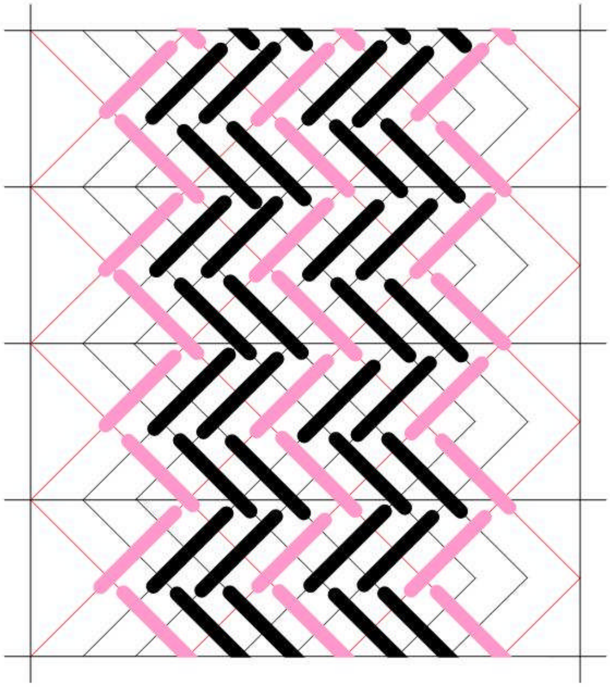
A further refinement to the rim surrounding the gap in the punctured sphere is possible. One can trim a complete rim off Nested Knot Grid. Actually, you can play this trick on any grid with a rim, but for Nested Knots the result is dramatic. I do not know of any customary name for this operation, but Neil Hood from Australia was the first I met studying these diagrams too. So, I called it Hooding.

The Symmetrical Nested Knot (4,5,12,2) consists of 4 components. Hooding the right-hand side rim once causes a single component grid to come forth. The trimmed right rim is indicated by means of dotted lines in Fig.8.

Epilogue

Here we have had some fun looking at bare bone Nested Knot Grids. Be stretched and go one step further. Try placing various coding-flavors on these structures and observe what happens. They are also a great basis for further interweaves.

Before closing off I would like to point out that we have mainly hunted the single stringers here. However, when your project requires some truly symmetrical structure you may need to abandon the single string demand. In Fig.9 we show the 3-component grid of (4,3,13,3), equipped with a 3-pass Herringbone Weave. Good luck!



3-pass HerringBone Weave on (4,3,13,3)

Fig.9

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From the Mailbagbag

Dan Lehman of sent in ViVirrginia article: “In these comments about a past *Knot News* 51:10 ‘The Fishermann Knot’ Percy Blandford presents images for bothth a loop knot and bend that are not what he inntetended. The left-hand Overhand component in iimages A & B is incorrectly oriented in that thee line around which it’s tied isn’t parallel with ththee Overhand’s ends – as it should be for a Fisheerrman’s Knot. Further, his image E for how to ccomplete the tying of the loop knot in the bigght is wrong; there should be no right-most twisst/crossing of the bight – only the initial crossingss shown as D. Thinking about his report that climmbbers dropped the use of the loop knot as a Midd dlleeman’s Knot, possibly on account of its vululnnerability to function qua noose if loaded on thhee end leading to the first (the left, in his iimmage’s case) Overhand, I wonder if the tyingg method he presents was so much the only oonne known by users that they thus didn’t think toto redress the nose/sliding problem by makkiing a third Overhand in the same part of the line as the right one (i.e., there would now be left-center-right such component Overhands, and the outer two would be in one side, the center in the other?!) . These Overhand components can actually be tied in any order, and if this had been known, it should’ve been an easy matter to preclude the noose effect by casting one more Overhand in the appropriate part. Although a knot with three Overhands would have been unusual, “new” – and thus apparently intimidating to conservative users! Such a knot has been recently advanced as an offset bend, for abseiling – by the IGKT’s Heinz Prohaska and also a German recreational rock climber, Jost Gudelius; it is yet to receive much recognition, let alone acceptance, although it should function well for the purpose.

The 4-Lead x 14 Bight Möbius Turk's Head

As explained by Geoffrey Budworth

August Ferdinand Möbius was a 19th century mathematician and astronomer who, in 1858, identified some of the extraordinary characteristics of an endless band of paper which had been cut, the ends given half a turn, and then rejoined. With a pencil, starting anywhere, draw a line along the center of the strip. Keep going until you arrive back where you began, and you will have drawn on *both* sides of the paper without lifting your hand. Alternatively, run a fingertip along the edge. The outcome will be similar. It is bizarre but – believe it - a Möbius band has only ONE side and ONE edge.

In February 1990, the Dutch-born Georg Schaake and British IGKT member Dr. John Turner, both at the University of Waikato, New Zealand, recorded informally their discovery of how to tie Möbius band Turk's Heads. Five months later they reported the event in *Knotting Matters* #32 with, as proof of their mastery of the mystique, illustrations of a 6-lead x 64-bight specimen as well as a wider 'Headhunter' (over two – under two weave) knot with 79 bights. [These two Turk's Heads are not as enormous as their dimensions suggest, because each Möbius Turk's Head – like the paper band – has only one edge, and so all the rim parts or bights can only be counted as one total.]

Schaake and Turner challenged IGKT members to tie, without any instructions from them, a 4L x 14B Möbius Turk's Head… and, by trial and error and discovery learning, several individuals did so. Charlie Smith (England) adapted his original technique for tying orthodox Turk's Heads around his upturned fingers – referred affectingly by our late Guild President Brian Field as '*the dead spider method*' – to the task. It is this process which is now described and illustrated.

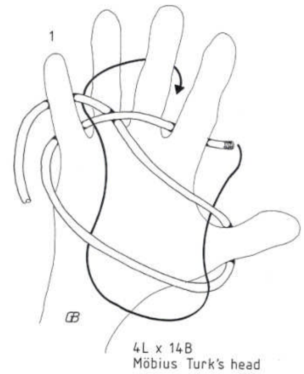

Fig 1. Arrange the cord around thumb and fingers, then interweave the working end going alternately U-O-U-O-U, as shown, to pass around the middle finger.

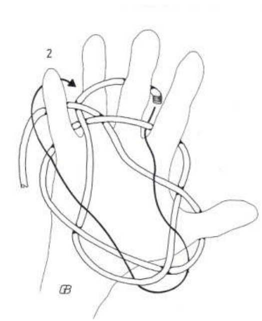

Fig 2. Continue to tuck the working end U-O-O-U-U-O-O-O-O to pass around the little finger.

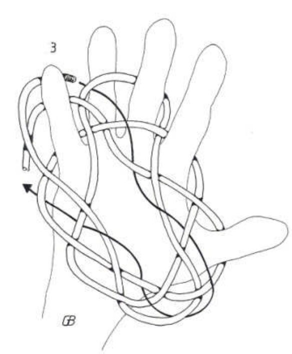

Fig 3. Seek out the ladder with a long locking tuck, going alternatively U-O-U-O-U-O-U-O-U-O-U (and then finally –U) so that the working end emerges close to the standing end.

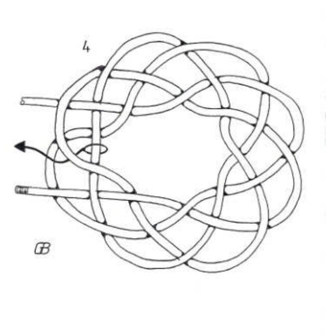

Fig 4. Remove the almost completed knot from the hand and withdraw the bight shown.

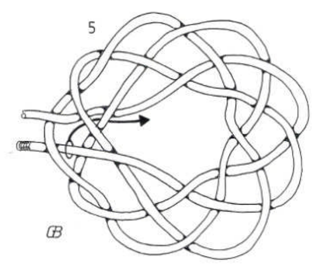

Fig 5. Withdraw the working end from beneath its final crossing point and tuck it alongside the standing part as shown.

Do not become discouraged if at this point your Turk's Head appears flawed.
Provided you have completed each stage exactly as shown, with regular over & under sequence, it will be correct. Remove some of the slack and then double the ply of the knot by following around the original lead, after which its true form should emerge.

Confessed a rope-worker named Fred
'It's self evident, I've always said,
That each knot *has a use…*
But I cannot deduce
What to do with a Möbius Turk's Head.

Like Fred, I too have always averred that knots – no matter how complicated and ornate, are all essentially practical; but even I cannot suggest what might be done with a Möbius Turk's Head. Europa Chang (UK – Essex Branch) made one into a brooch from which she hangs her reading glasses.
Any more ideas?

Further Reading

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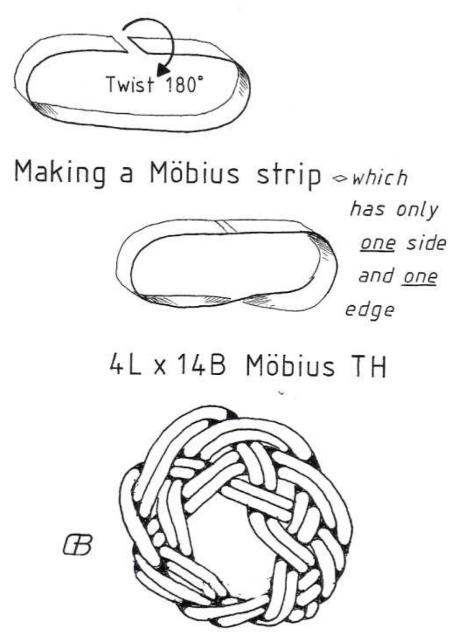

Words from a Recycled President

Hi! "Once more unto the breach, dear friends, once more…" I volunteered to take on the post of being your President for another stint, in the hopes that someone would get riled up and say, "Not again!" and put their own name forward. That was not to be however, and so I am delighted to once again represent you in our official dealings, whatever they may be! Let me tell you of some of the plans that your Board and Officers have for the next tow years, and if you have an opinion you would like to share about those plans (or anything else you want to get off your chest) drop me a line or call, write or speak to any of our other Officers and Board Members.

First, let me tell you who the fine people are with whom I have the pleasure of serving the two years. Mr. Jimmy Ray Williams has stepped up to be the new Secretary/Treasurer. This is a challenging position, as past Secretaries can attest! At his side is Mr. Jose Hernandez-Juviel, stepping up to the plate for a second term as Librarian. Our Board Members are Mr. Joe Schmidbauer, Mr. Roy Chapman, Mr. Patrick Ducey and Mr. Brian Kidd. If you are not familiar with any of these names, please examine your roster or ask Jimmy Ray to email you their contact information.

The first effort that your Officers and Board are undertaking is that of making our web pages more accessible to all. I have asked two of our members, with special skills in languages other than English, to prepare translations of the web site for people who do not read or speak English, or who cannot understand it when browsing the web for information about knotting. Their special talents are in the languages of French and Spanish, and they have been very active in putting together translated pages "idiomized" rather then plainly translated word-for-word, so that readers in those languages may feel more at ease when reading the web information.

The next effort we shall be undertaking is to poll you, our existing members, about your wishes for activities that our Branch should or should not undertake, and how you would like to see activities that help bring our 71 members together from our far-flung corners of the world. Your input is vital to this effort, so please stand by to complete that information and pass it on to us, because we really want to know!

The third effort that we are going to undertake is to find a venue and suitable celebration for our tenth anniversary next year. Tentatively we are thinking about San Diego as being accessible to all who travel by air, sea, train or motor car. Your input again is sought so that we can make this a gathering of knotters who want to be with and learn or teach their fellow knotters. We will identify travel information; lodgings (including camp sites); venue locales for the weekend and a dinner location for the Friday night of our Annual General Meeting. We also are planning a Special Anniversary Edition of *Knot News* for our readers. If you want to have something included, better hurry and get it in!

Thank you as always for your continuing faith and trust in our efforts at running the Branch – we are on your side and so we want to hear from you. Pull up a fence, as they say, and lean over – we are listening!

Lindsey Philpott, President IGKT-PAB

Well, folks, I guess this proves the old saying, "Be careful what you ask for, because you may get it." I volunteered to try my hand at Secretary/Treasurer, and now I are it! And, I'm surprised to learn that there is WORK involved! WHO KNEW! (Work is a 4 letter word!)

First thing I would like to do is to get an update from everyone on your statistics: your name, address and phone number (or numbers). If you have access to the internet, please include an email address and your personal web site (if you have one). Even if I have the correct information, I'd like verification, please.

I would also like to have a synopsis of your knotting skills and passions. What got you started in this tangled web we weave? No need for a great amount of detail. Just a little bit of your personal knotting experiences. And perhaps a little note on what you expect as a result of being a member of the IGKT –Pacific Americas Branch. You can send this info to me via an email

Jimmy Ray Williams, Secretary/Treasurer IGKT-PAB