Knot News

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Notes on Knots

On the Work of Henry North Grant Bushby

by Pieter van de Griend

The Virginian woodlands at Newport News conceal the buildings of a maritime museum. A huge phosphor-bronze ship's propeller at the entrance gives a cue to what these premises harbor. Little known to the knot world at large is the fact that the Mariner's Museum research library's private collection holds some of the biggest treasures any knot researcher could ever dream to stumble upon. Not only do they hold the near-complete set of *original* drawings of Clifford Ashley's monumental *Book of Knots* [3], but also a unique manuscript by a Henry North Grant Bushby, consisting of some 8 small, thick, densely scribbled notebooks on knots! In 1958 his daughter, Miss Dorothy Bushby, bequeathed them to the museum. My attention was drawn to this manuscript by a Swedish knotologist's letter some 18 years ago [14]. The knot world has a lot of exciting obscure sources, but in my humble opinion, the Bushby Manuscript tops the bill. I think it ranks higher than, for example, the elusive 1919 Taber Paper on Square Turk's Head Knots [3, #1321].

During a long period Henry North Grant Bushby wrote down his observations and ideas about knots in those knot books. The earliest notes among these 2,000 pages date from spring 1902. The latest are less unambiguous, but the library's card index dates them to be from around 1926. This lavishly illustrated set of notebooks, which Bushby himself titled *Notes on Knots*, was never published. Judging from several notes in the manuscript Bushby intended to turn his thoughts into print. Sadly, this never materialized. Moreover, aside from his address, somewhere in Hyde Park London, and a single article he wrote in 1902 about an agreement between Imperial Japan and the United Kingdom, not much is know about this fascinating knot researcher from Herefordshire [7].

During the summer of 1997 I had the rare pleasure to spend some time in the Mariner Museum's Research Library to study these eight marvelous books. Due to that study this article arose. In this paper I want to give an indication of the ideas that formed the background for much of Bushby's research and which influenced his perception of the knot world. Ideas which undeniably radiate from each and every page. After that I want to give an indication of some of the many subjects he researched.

The first pages of Volume I list the many sources Bushby refers to throughout his work. Between those sometime esoteric sources, you encounter four names no pragmatic knotter would ever expect to find: Listing, Tait, Kirkman and Little. Those names tell you everything about the perspective Bushby wanted to implant on his knot study. Namely that of a Victorian Gentleman with a strong interest in the current state of affairs of scientific research.

If Henry North Grant Bushby really would have succeeded in publishing his manuscript, then I suspect he would have started his knot story with Carl Friedrich Gauss (1777-1855). For that reason this story will commence at that titan of science. From pages, dated 1794, among the Grauss heritage, it appears that he was interested in pragmatical knots at an early point in his scientific career [25]. Several years later

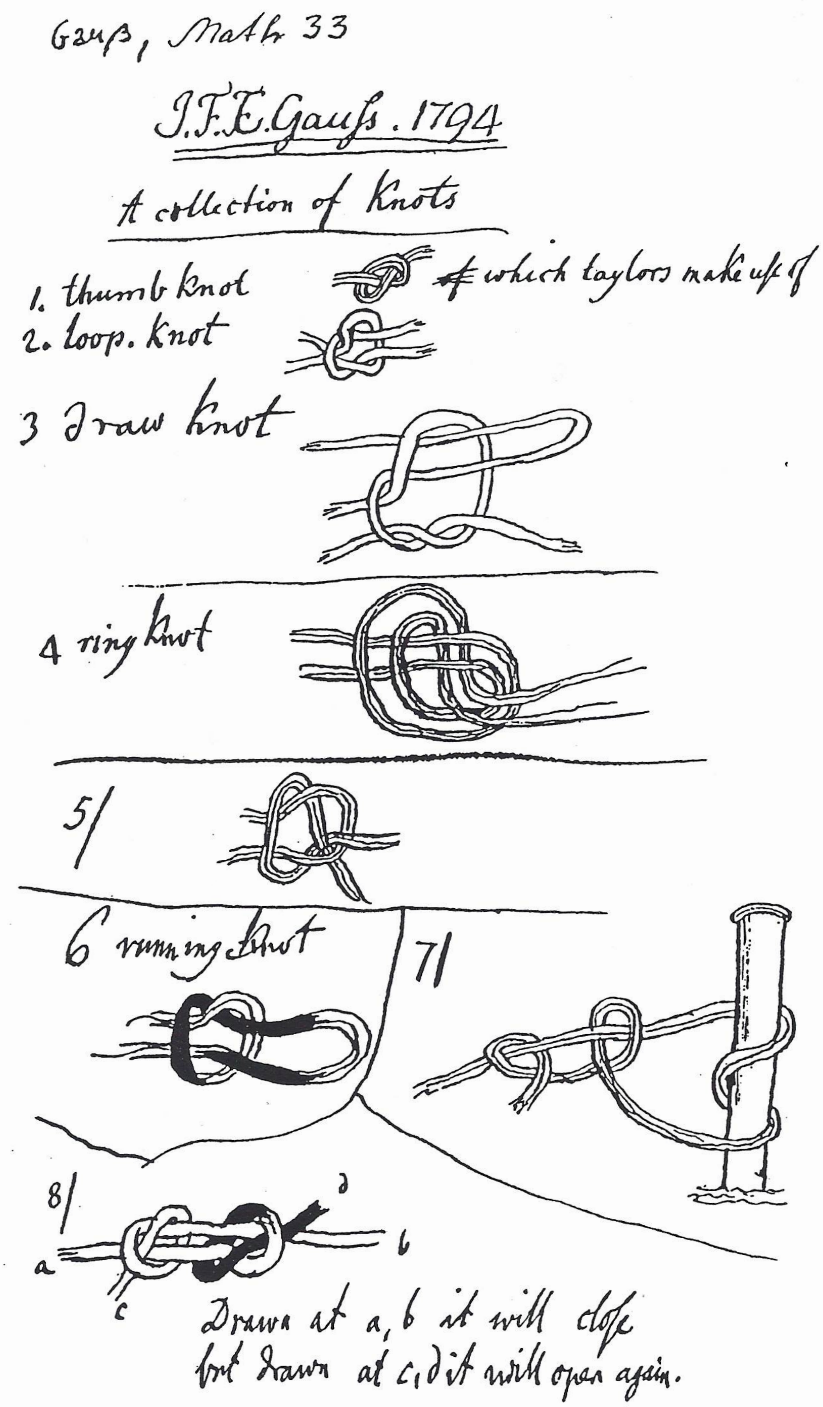


Fig.1 One of Carl Friedrich Gauß's Knot pages

this impressive German genius in the Natural Sciences had struck knots again. This time in his studies into electrodynamics [11]. He realized that knots were a difficult subject, worthy of study, and with which mathematics had barely occupied itself. There was not even a name for that part of mathematics which should study the knot-subject. Later it would be called topology. Nowadays topology is concerned with *invariant* properties of geometric objects, but in Gauss' day the world was not that far yet. Gauss got one of his students, Johann Benedict Listing, to delve into the field. After some 7 years Listing published a part of his findings. His booklet *"Vorstudier zur topologie"* would remain the sole source on mathematical knot theory for a long time. [18].

Listing considered knots as closed, though knotted, curves in space. Pragmatic knotters ferociously reject such objects being called knots. Their main argument is that such knots have no ends and no beginnings. Those ends are deliberately fused to ensure the essential knotted part to remain "knotted". A knot in some physical medium merely exists as a knot because certain parts of the tangled structure prevent other parts from untying themselves. From this interplay of forces and restrictions a knot may emerge. For pragmatical knotters different knots appear different for the simple reason that the ends are not allowed to get into the play. If they would be permitted to participate, say if the knot were tied in a very smooth material, all knots would be identical (to a straight line). Why should one want to prevent the ends participating in the dynamic manipulation process a knot may be subjected to? Remember what topology was about? The *invariant* properties of geometric objects, i.e. properties which never change. By letting the ends participate the nature of the knot may change and thus also its invariant geometric properties. Hmmm... Mathematics solved this problem by fusing the ends in such a way that a knotted grommet emerges. Further idealizations bring you to a smooth closed curve. In Fig. 2 you can see how a Reef Knot in rope gets its end fused and eventually ends up as a mathematical knot diagram. Note that this knot consists of one component. As will be obvious, the number of components in a knot is an example of an invariant knot property. No matter how much you tangle and twist your knot, its number of components will always remain the same.

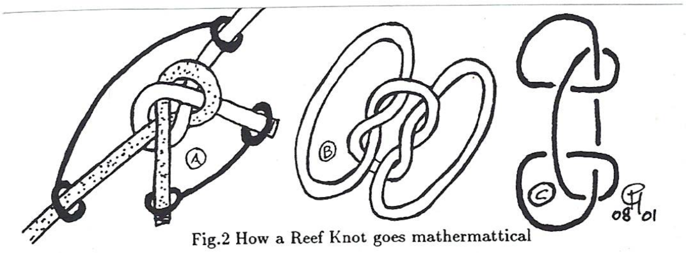


Fig.2 How a Reef Knot goes mathermattical

To distinguish one mathematical knot from the other Listing introduced the concept of an "invariant". That is a mathematical expression which should take on different values for every pair of distinct knots and which should take on identical values for any pair of identical knots. The first part is obvious: if you have two structurally different knots, then you want two distinct values for your variant. If you have two identical knots and you start deforming one of them into something what *appears* different from its non-distorted cousin, the resulting invariant values should be the same. Of course the values these invariants obtain must be calculated from the respective knot diagrams.

How did Listing assign his invariant knot diagram? First he considered so-called alternating knot-diagrams, those are diagrams in which the crossings *alternate* Over 1 Under 1 when you trace the knot. Next he assigned a token, in this case one of the symbols λ and δ, to every region in the knot-diagram (including the unbounded one). Regions of like token were not allowed to neighbor each other, except at crossing. Then he would count the number of sides in each region belonging to a token and place that in the token's exponent. Finally he summed all like tokens of equal exponent and put them in an expression surrounded by curly braces. In Fig. 3 this is shown for a specific knot. Note that there are two δ-regions of 4 sides and two δ-regions of 3 sides. Likewise there is one λ-region of 4 sides, two λ-regions of 3 sides and two λ-regions of 2 sides. Listing called his type of invariant a Complexions-symbol.

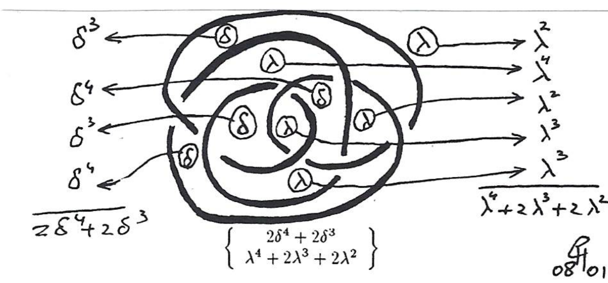
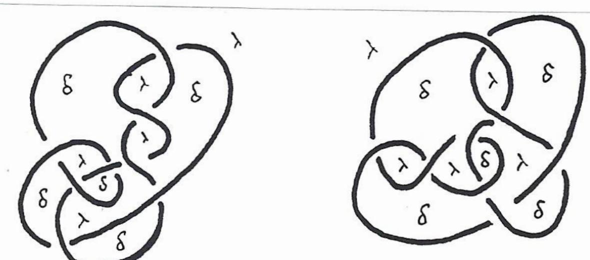


Fig.3 Example of how to calculate Listing's Complexion-Symbol

If you have never seen anything from mathematical research in your life before, then Listing's procedure may appear a bit haphazard. There exist arguments why his concoction should mirror invariant knot-diagram properties, but they are well beyond the scope of this article [13]. Sadly, however, the complexion-symbol died because it is not any powerful invariant. Much like the number of components in a knot does not distinguish all different knots of one component, the Complexions-symbol fails on certain knots too. Consider the distinct knot-pair in Fig. 4. You cannot tangle or twist one knot into the other, yet they have equal complexion-symbol [22], [23]. We can only conclude that this invariant has limited discerning capabilities.



$$\left\{ \begin{array}{c} \delta^5+\delta^4+\delta^3+2\delta^2 \\ \lambda^5+\lambda^4+\lambda^3+2\lambda^2 \end{array} \right\}$$

Fig.4 Two distinct knots with identical Listing Complexion-Symbol

Although we just seen Listing's invariant fail, much of the research which takes place within modern knot theory is still heavily concerned with invariants, but not any longer of the kind shown [15].

The story of mathematical knots did not stop with Listing. After a small break it continued around 1870. During their researches into physical phenomena two Scottish scientists had stumbled onto knots. Sir William Thompson, better known as Lord Kelvin, and Peter Guthrie Tait thought they could explain all properties of chemical elements in terms of knots tied in a hypothetical medium called *aether*. Aether was the medium through which the scientific community, way back then, believed electro magnetic waves were propagated. This aether medium was assumed to permeate space and fill the interstices between particles of matter. It required a few properties which it simply had to posses in order for it to be somewhat useful for that purpose. One of them was that it had to consist of little tubes. Their latin name is *vortex*, and *vortices* as its plural. For this reason Tait and Thompson's ideas were known as Vortex Theory. It seeked to explain the stability of atoms and to classify the elements of the well-known Periodic Chart from chemistry. Since they saw the elements as distinct little knots in the aether vortices, it became obvious that they could just as well classify knots. As knowledge about knots was virtually non-existent Tait started his pioneer study into knots. His goal was to find a way to classify them. Within scientific circles the Vortex Theory was taken very serious for quite some time. To begin with there was no counter theory and many prominent scientists were proponents. Only at the beginning of the 20th century did Quantum Theory replace Vortex Theory. However, before that move took place Peter Tait had done considerable original research into knots. Some of it as a joint venture with Rev. Kirkman and C.N. Little [19], [22], [23].

One of Tait's group's greatest achievements was classifying simple knotted structures by their least possible number of crossings. They identified and classified the majority of all knots of 10 or less crossings. A part of their results, all knots with 5 or less crossings, is shown below.

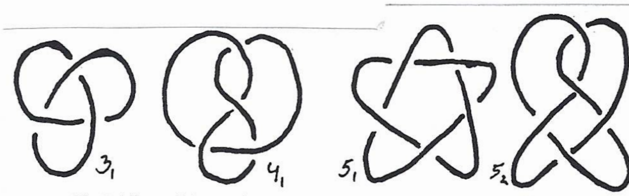


Fig.5 All possible mathermaticcal knots with 5 or less crossings.

This classification problem is a lot tougher than it may seem, because knots have a very treacherous property. If we restrict ourselves to mathematical knots, then you can make any knot appear different by merely adding twists and tucks. Your newly created knot will *appear* more complex than the knot from which you started, but it is still the same knot. You need merely reverse the process to reach your starting point again. This property of knots is called *isotopy*. In a sense, the knot possesses knowledge of how it should look in its simplest form. Simply because that (invariant) knowledge was trapped when the ends were fused. If you'd allow the ends to participate in the tucking and tangling process, like in trambling, it would be a simple thing to untie the knot. There would be nothing left to help the knot remember who it was. With mathematical knots such amnesia will not occur.

We have spoken a lot about mathematical knots. Let's now return to Bushby who was much taken by those early mathematical knot researchers who had given him the tools to work on some special thoughts concerning pragmatic and mathematical knots. Throughout all of his work Bushby does what knotters are best at: structure recognition. He would consider pragmatical knot(s) and notice some collective theme. On the other hand he elaborated and illustrated variations to that theme by means of isotoping the corresponding mathematical knot(s). To distinguish his mathematical knots he had notational shorthand called *"geometric form"*. It was a description which indicated the number of crossings and bights of a knot laid down in a special type of knot-diagram. In a sense it was a rudimentary invariant. Bushby tried hard to extract the number of components from his geometric form. As this is a tough exercise he later let the geometric form incorporate the number of parts the mathematical knot possessed when considered as a braid. Bushby was already aware of the method which enables one to transform any mathematical knot into a combed braid which fits around a cylinder. A method which later became known as Alexander's Method, in honor of the great American mathematician James W. Alexander [1], [5]. In Fig. 6 you can see how he subjected the Weaver Knot and the Timber Hitch to his transformations.

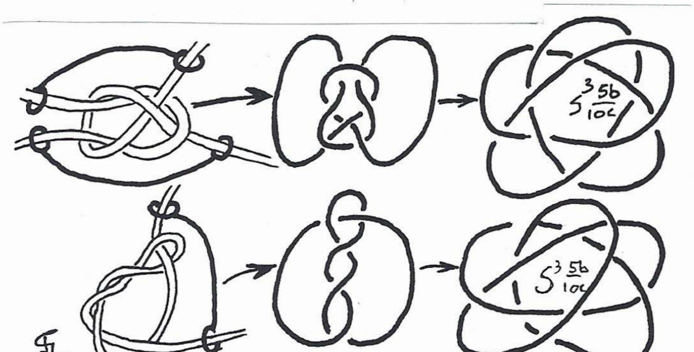


Fig. 6 Two examples of Bushby geometric form $S^3\frac{5b}{10c}$.

Using his geometric form he tried to study the structures very reminiscent of Turk's Head Knots. During his investigations into what he called *"Plait Knots"*, he stumbled upon the Law of the Greatest Common Divisor in braids. This law states that a Turk's Head Knot with p parts and b bights will have a number of components which equals the greatest common divisor of p and b. For example, a Turk's Head Knot of 23 parts and 17 bights will be single stranded, as the greatest common divisor of the primes 17 and 23 equals 1. On the other hand, a Turk's Head Knot of 26 parts and 39 bights will require 13 strands to make (without cheating), as 13 is the greatest number which divides both 26 and 39. In the knotting literature the law was first mentioned some thirty years later, namely around 1930-45 [3, p233], [12, p440]. Although Bushby's proof for the correctness of the law is not very convincing, it is the first recorded attempt at proving it. Generally accepted proofs first appeared in print around 1980-90 [20], [24].

Investigating alternating Turk's Head Knots and realizing the validity of the Law of the Greatest Common Divisor, led Bushby to have a go at non-alternating and less regular knots, where he also believed to see certain laws. His attempts at proving them were no success.

As said, Bushby used the mathematical knot diagrams to extend pragmatical knots. Using their isotopical properties he could show that the Sheet Bend, Bowline and Pile Hitch [3, #1815] are interrelated. The Pile Hitch was not acknowledged in the mariner's knotting literature of Bushby's days. It had no name and Bushby thus saw it as his invention.

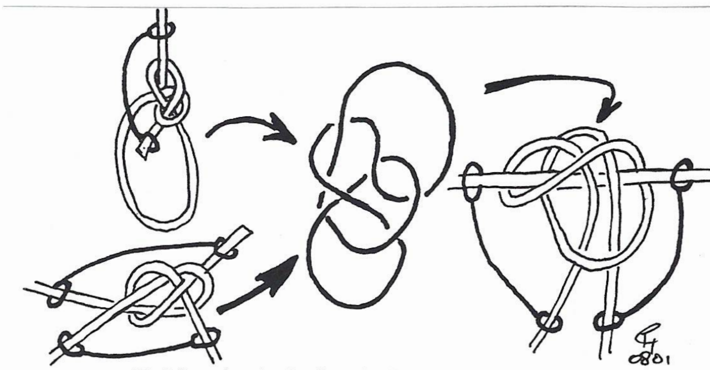


Fig.7 Isotoping the Bowline, the Sheet Bend and the Pile Hitch.

Bushby gave the Sheet Bend Structure a lot of thought. He illustrated many ways to tie a Sheet bend and considered the capsizing of a Slip Knot as *the* way knotters may have discovered the Sheet Bend. One of his many methods is given below.

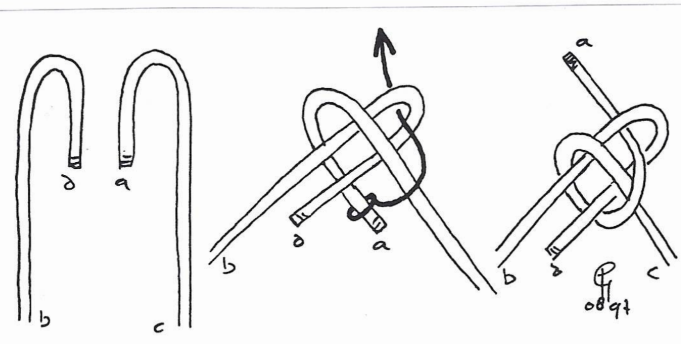


Fig. 8 One of Bushby's methods to produce the Sheet Bend

As was expected, many of Bushby's sources were mariner-related [2], [9], [16], [17], [21]. Like many of his fellow knot researchers at the beginning of last century he also frequently resulted to encyclopedias such as *Encyclopedia Britannica* and *Chamber's Encyclopedia* and dictionaries such as *Murray's Dictionary* and *Ogilvie's Dictionary*. Also Tom Bowling and Joseph Burgess were also consulted and referred to [4], [6]. In that light it is interesting to find notes and an illustration of the Gunner Knot. Ironically enough referring to Tom Bowling [8]. His illustration and accompanying text is given below.

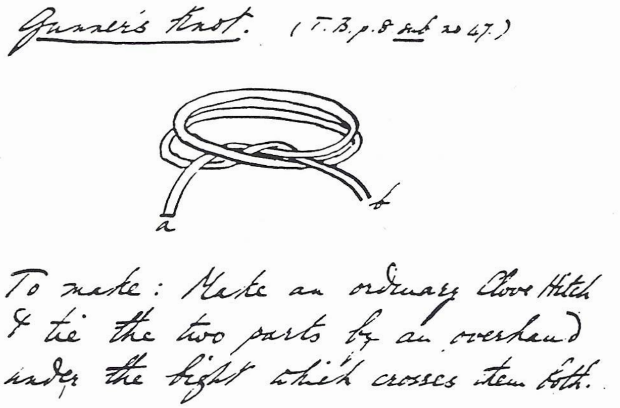


Fig.9 Reproduction from the Bushby Manuscript.
Courtesy Mariner's Museum, Newport News.

There is no doubt that Henry North Grant Bushby was a man who was fascinated by structure and its transformational abilities. He dissected any knot pattern he came across and cooked up variations to the theme. Ways the Lowestoft fisherman mended their nets a century ago, were critically examined and compared. He was fascinated by knots birds tie and apparently conducted some letter writing with anybody bold enough to write anything about knots. It is a great pity that this colorful and highly original piece of work discussing hundreds of knot types never made it to the printer shop. Undoubtedly its publication would have changed the face of knotting.

This article greatly benefited from the enthusiastic cooperation of the friendly staff at the Research Library of the Mariner's Museum; 100 Museum Drive; Newport news, Virginia 23606-3759 USA. I gratefully acknowledge their help and permission to publish parts from the Bushby manuscript.

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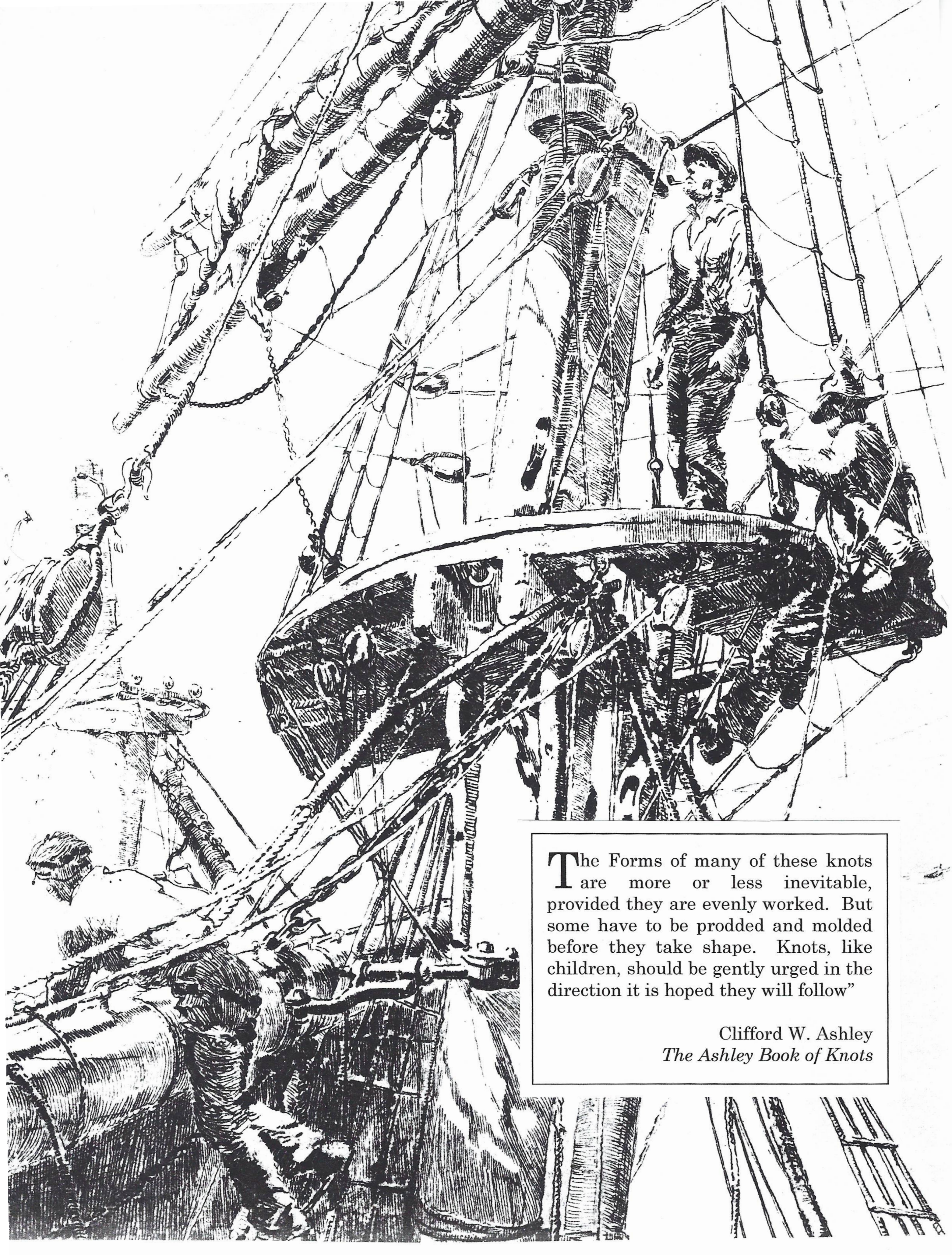

The Forms of many of these knots are more or less inevitable, provided they are evenly worked. But some have to be prodded and molded before they take shape. Knots, like children, should be gently urged in the direction it is hoped they will follow"

Clifford W. Ashley
The Ashley Book of Knots

Pacific Americas Branch Schedule

Our Monthly meetings are held on the Second Tuesday of each month from September through June. They are held at the Los Angeles Maritime Institute in San Pedro, California. This is right next door to the Los Angeles Maritime Museum at the foot of Sixth Street at Berth 84. Meetings run from 7:00 to 9:00 PM.

The meeting dates for the remainder of 2001 are November 13th and December 11th.

All members are welcome to attend and everyone is invited to bring a piece of ropework you have created, a tool or technique you find useful or just some creative tips for your fellow knotters.

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