KNOT NEWS

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Knot Ties and Random Walks

by Thomas Fink and Yong Mao

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The simplest of conventional tie knots, the Four-in-Hand, has its origins in late nineteenth-century England. The Duke of Windsor, after abdicating in 1936, has been credited with introducing what is now known as the Windsor knot, whence its smaller derivative, the Half-Windsor, evolved. More recently, in 1989, the Pratt knot was revealed on the front page of the *New York Times*, the first new knot to appear in 50 years.

Rather than wait another half-century for the next sartorial advance, here we present a more formal approach. We introduce a mathematical model of tie knots and provide a map between tie knots and persistent random walks on a triangular lattice. We classify knots according to their size and shape and quantify the number of knots in each class. The optimal knot in a class is selected by the proposed aesthetic conditions of symmetry and balance. Of the 85 knots which can be tied with a conventional tie, we recover the four knots in widespread use and introduce six new aesthetic ones.

A tie knot is initiated by bringing the wide (active) end to the left and either over or under the narrow (passive) end, dividing the space into right, centre and left (*R*,*C*,*L*) regions (Fig. 1a). The knot is continued by subsequent half-turns, or moves, of the active end from one region to another (Fig. 1b) such that its direction alternates between out of the shirt and into the shirt ($\odot$, $\otimes$). To complete a knot, the active end must be wrapped from the right (left) over the front to the left (right), underneath to the centre and finally through (denoted *T* but not considered a move) the front loop just made.

Elements of the move set $\{R_\odot, R_\otimes, C_\odot, C_\otimes, L_\odot, L_\otimes\}$ designate the moves necessary to place the active end into the corresponding region and direction. We can then define a tie knot as a sequence of moves initiated by $L_\otimes$ or $L_\odot$ and terminating with the subsequence $R_\odot L_\otimes C_\odot T$ or $L_\odot R_\otimes C_\odot T$. The sequence is constrained such that no two consecutive moves indicate the same region or direction.

We represent knot sequences as random walks on a triangular lattice (Fig. 1c). The axes r,c,l correspond to the three move regions R,C,L and the unit vectors $\hat{\mathbf{r}}$, $\hat{\mathbf{c}}$, $\hat{\mathbf{l}}$ represent the corresponding moves; we omit the directional notation $\odot$, $\otimes$ and the terminal action T. Since all knot sequences end with $C_\odot$ and alternate between $\odot$ and $\otimes$, all knots of odd numbers of moves begin with $L_\odot$ while those of even numbers of moves begin with $L_\otimes$. Our simplified random walk notation is thus unique.

The size of a knot, and the primary parameter by which we classify it, is the number of moves in the knot sequence, denoted by the half-winding number h. The initial and terminal sequences dictate that the smallest knot be given by the sequence $L_\odot R_\otimes C_\odot T$, with h=3. Practical (*viz.*, the finite length of the tie) as well as aesthetic considerations suggest an upper bound on knot size; we limit our exact results to half-winding number $h \leq 9$.

The number of knots as a function of size, $K(h)$, corresponds to the number of walks of length h beginning with $\hat{\mathbf{l}}$ and ending with $\hat{\mathbf{r}}\,\hat{\mathbf{l}}\,\hat{\mathbf{c}}$ or $\hat{\mathbf{l}}\,\hat{\mathbf{r}}\,\hat{\mathbf{c}}$. It may be written

$$K(h) = \tfrac{1}{3}\left(2^{h-2} - (-1)^{h-2}\right), \qquad (1)$$

where $K(1) = 0$, and the total number of knots is $\sum_{i=1}^{9} K(i) = 85$.

The shape of a knot depends on the number of right, centre and left moves in the tie sequence. Since symmetry dictates an equal number of right and left moves (see below), knot shape is characterised by the number of centre moves γ. We use it to classify knots of equal size h; knots with identical h and γ belong to the same class. While a large centre fraction $\frac{\gamma}{h}$ indicates a broad knot (*e.g.*, the Windsor) and a small centre fraction suggests a narrow one (*e.g.*, the Four-in-Hand), not all centre fractions allow aesthetic knots. We consequently limit our attention to $\frac{\gamma}{h} \in \left[\frac{1}{4}, \frac{1}{2}\right]$.

The number of knots in a class, $K(h,\gamma)$, is equivalent to the number of walks of length h satisfying the boundary conditions and containing γ steps $\hat{\mathbf{c}}$; it appears as

$$K(h,\gamma) = 2^{\gamma-1}\binom{h-\gamma-2}{\gamma-1}. \qquad (2)$$

The symmetry of a knot, our first aesthetic constraint, is the number of moves to the right minus the number of moves to the left, *i.e.*,

$$s = \sum_{i=1}^{h} x_i, \qquad (3)$$

where $x_i = 1$ if the ith step is $\hat{r}$, -1 if the ith step is $\hat{l}$ and 0 otherwise. Since asymmetrical knots disrupt the bilateral symmetry of man, we limit our attention to the most symmetric knots from each class, *i.e.*, those which minimise s.

Whereas the centre number γ and the symmetry s tell us the move composition of a knot, balance relates to the distribution of these moves; it corresponds to the extent to which the moves are well mixed. A balanced knot is tightly bound and keeps its shape. We use it as our second aesthetic constraint. The balance b may be expressed

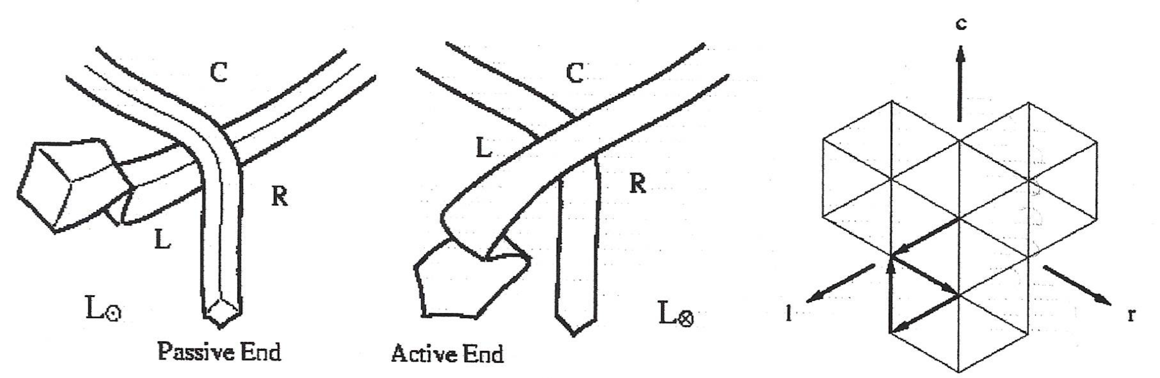
$$b = \frac{1}{2} \sum_{i=2}^{h-1} |\omega_i - \omega_{i-1}|, \qquad (4)$$

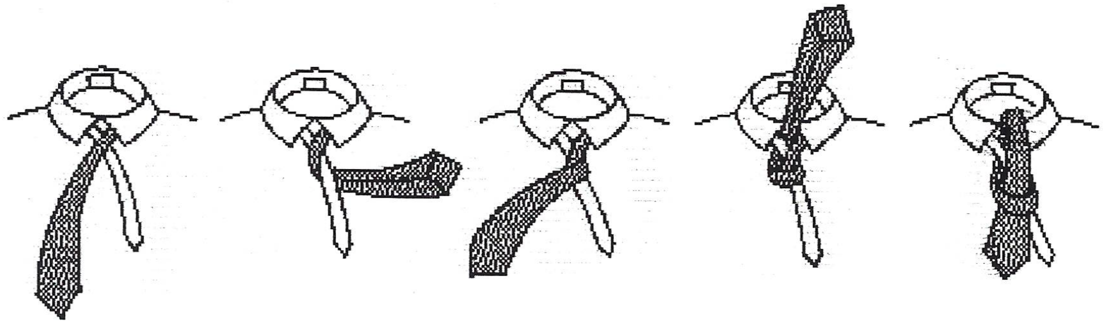
where σ_i represents the ith step of the walk and the winding direction $\omega_i(\sigma_i, \sigma_{i+1})$ is equal to 1 if the transition from σ_i to σ_{i+1} is, say, clockwise and -1 otherwise. Of those knots which are optimally symmetric, we desire that knot which minimises b.

The ten canonical knot classes $\{h, \gamma\}$ and the corresponding most aesthetic knots are listed in Table 1. The four named knots are the only ones, to our knowledge, to have received widespread attention, either published or through tradition. Unnamed knots are hereby introduced by the authors.

The first four columns describe the knot class $\{h, \gamma\}$, while the remainder relate to the corresponding most aesthetic knot. The center fraction $\frac{\gamma}{h}$ provides a guide to knot shape, the higher fractions corresponding to broader knots; it, along with the size h, should be used in selecting a knot.

Certain readers may observe the use of knots whose sequences are equivalent to those shown in Table 1 apart from transpositions of $\hat{r}, \hat{l}$ groups, for instance, the use of $L_\otimes R_\odot C_\otimes R_\odot L_\otimes C_\odot T$ in place of the Half-Windsor (T. P. Harte and L. S. G. E. Howard, personal communication); some will argue that this *is* the Half-Windsor. Such ambiguity follows from the variable width of conventional ties -- the earliest ties were uniformly wide. This makes some transpositions arguably favourable, namely the last $\hat{r}, \hat{l}$ group in the knots $\{5, 2\}$, $\{6, 2\}$, $\{7, 2\}$, $\{8, 3\}$, $\{9, 3\}$ in Table 1. We do not attempt to distinguish between these knots and their counterparts; this much we leave to the sartorial discretion of the reader.





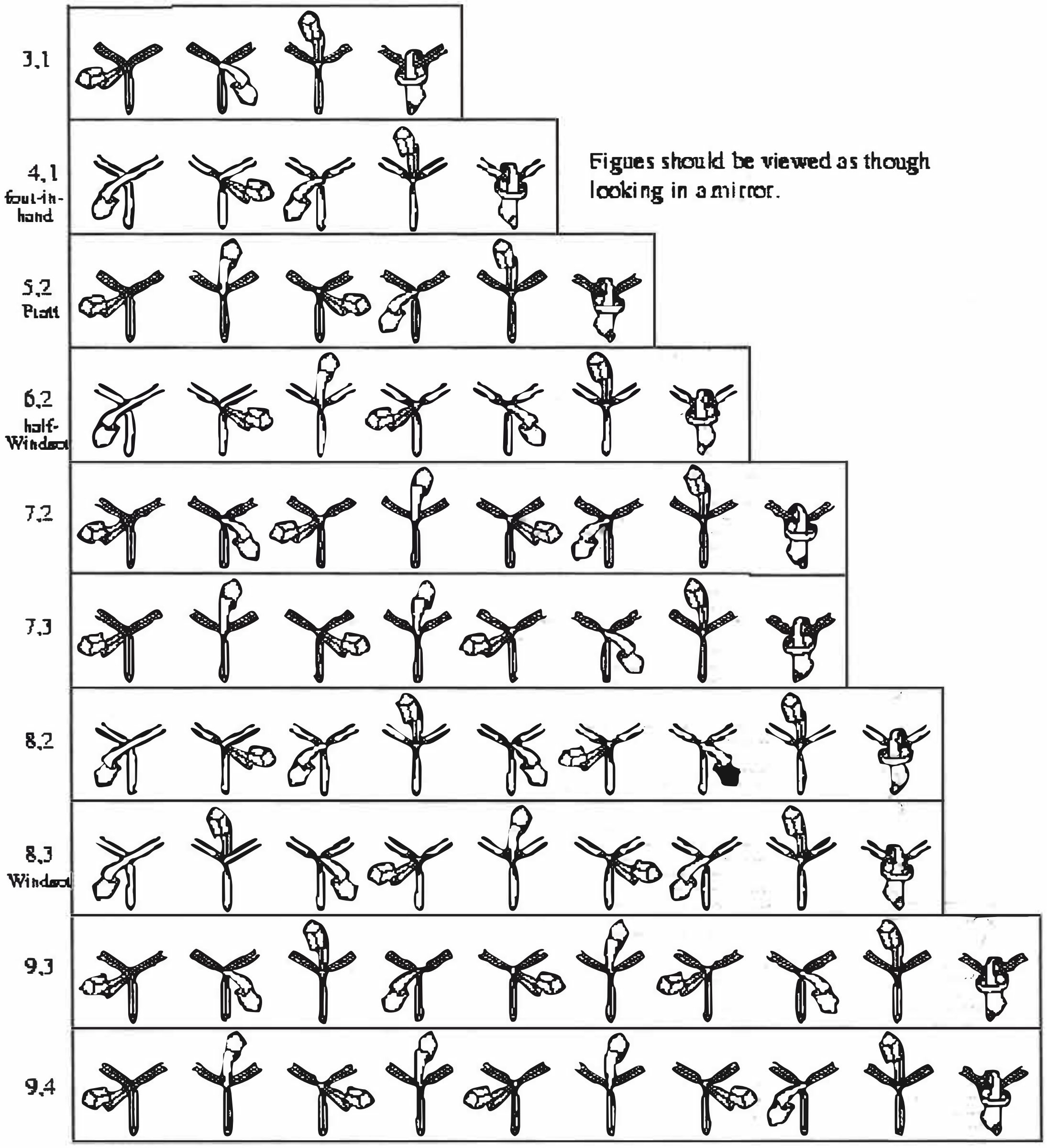
{ All diagrams are drawn in the frame of reference of the mirror image of the actual tie. **(top left)** The two ways of beginning a knot, $L_{\ominus}$ and $L_{\otimes}$. For knots beginning with $L_{\ominus}$, the tie must begin inside out. **(bottom)** The Four-in-Hand, denoted by the sequence $L_{\otimes}R_{\ominus}L_{\otimes}C_{\ominus}T$. **(top right)** A tie knot may be represented by a persistent random walk on a triangular lattice. Shown here is the Four-in-Hand, indicated by the walk $\hat{l}\,\hat{r}\,\hat{l}\,\hat{c}$.

h	γ	γ/h	$K(h,\gamma)$	s	b	Name	Sequence
3	1	0.33	1	0	0		$L_{\ominus}R_{\otimes}C_{\ominus}T$
4	1	0.25	1	-1	1	Four-in-Hand	$L_{\otimes}R_{\ominus}L_{\otimes}C_{\ominus}T$
5	2	0.40	2	-1	0	Pratt Knot	$L_{\ominus}C_{\otimes}R_{\ominus}L_{\otimes}C_{\ominus}T$
6	2	0.33	4	0	0	Half-Windsor	$L_{\otimes}R_{\ominus}C_{\otimes}L_{\ominus}R_{\otimes}C_{\ominus}T$
7	2	0.29	6	-1	1		$L_{\ominus}R_{\otimes}L_{\ominus}C_{\otimes}R_{\ominus}L_{\otimes}C_{\ominus}T$
7	3	0.43	4	0	1		$L_{\ominus}C_{\otimes}R_{\ominus}C_{\otimes}L_{\ominus}R_{\otimes}C_{\ominus}T$
8	2	0.25	8	0	2		$L_{\otimes}R_{\ominus}L_{\otimes}C_{\ominus}R_{\otimes}L_{\ominus}R_{\otimes}C_{\ominus}T$
8	3	0.38	12	-1	0	Windsor	$L_{\otimes}C_{\ominus}R_{\otimes}L_{\ominus}C_{\otimes}R_{\ominus}L_{\otimes}C_{\ominus}T$
9	3	0.33	24	0	0		$L_{\ominus}R_{\otimes}C_{\ominus}L_{\otimes}R_{\ominus}C_{\otimes}L_{\ominus}R_{\otimes}C_{\ominus}T$
9	4	0.44	8	-1	2		$L_{\ominus}C_{\otimes}R_{\ominus}C_{\otimes}L_{\ominus}C_{\otimes}R_{\ominus}L_{\otimes}C_{\ominus}T$

Aesthetic tie knots, characterised, from left, by half-winding number h, centre number γ, centre fraction $\frac{\gamma}{h}$, knots per class $K(h,\gamma)$, symmetry s, balance b, name and sequence.

Aesthetic Tie Knots

Thomas M.A. Fink



Figues should be viewed as though looking in a mirror.

Branch Bits

Tom Hall of Bastrop, Texas sent me this letter:

In KN #16 there was not much on how to tie any knots. I think 'how-to' articles make a newsletter more interesting. So, here are some drawings of two knots I have been helping a friend to tie. He has a yacht and wanted to tie knots on the ships wheel. I first showed him the Boat Steering Wheel Knot that was printed in KN #12. He thought a longer knot that had less bight would be better. We tried different sizes and came up with the 12 Part 7 Bight 1 Pass Knot with a 6 bight hole.

I told him we should do a 'T' knot, so the know would go down the center spoke. He thought it might look good to have a knot with a hole in it at all of the spokes except the center spoke where you have a 'T' knot. To look the same as the other knots, I drew up a 12 Part 7 Bight by 8 Part 4 Bight 1 pass 'T' Knot.

I hope some of you will enjoy tying these knots.

12 Part 7 Bight by 8 Part 4 Bight 1 pass

"T" Knot

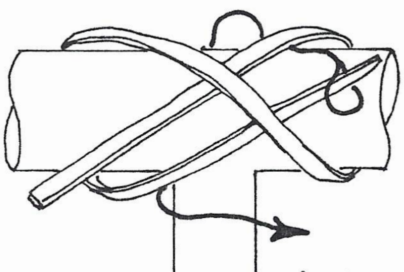


fig. 1

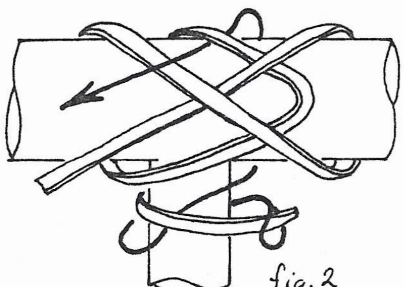


fig. 2

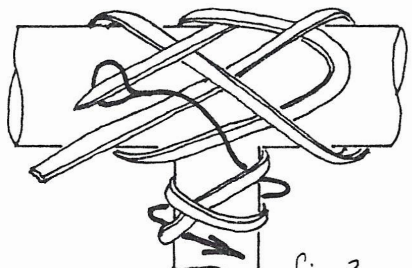


fig. 3

1. Free run
2. O
3. U
4. U
5. U2 O
6. O2 U2

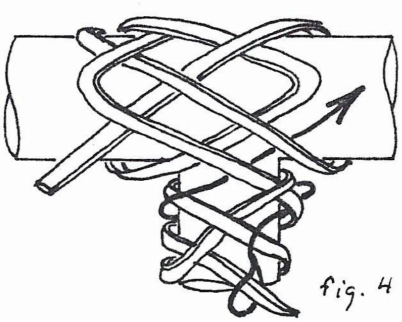


fig. 4

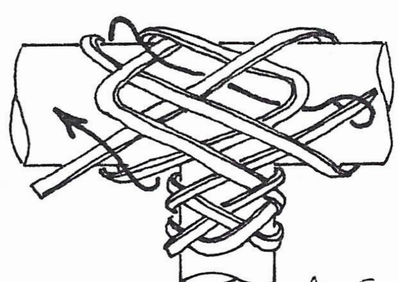


fig. 5

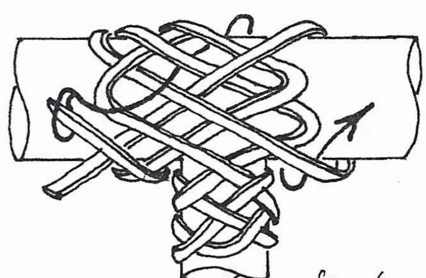


fig. 6

7. O U O U2
8. U2 O3
9. O2 U3

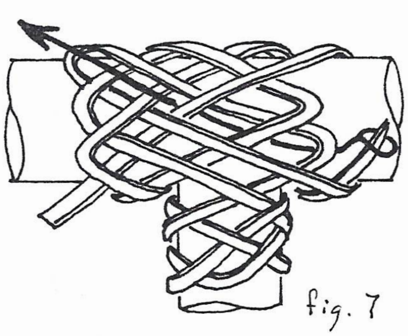


fig. 7

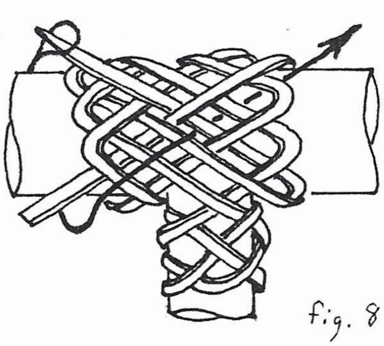


fig. 8

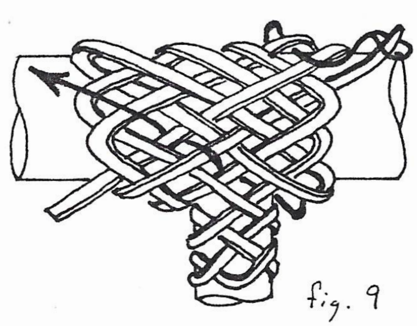


fig. 9

10. U3 O3
11. O3 U3
12. O U3 O U O2

12P/7B by 8P/4B 1 pass "T" Knot
(Cont.)

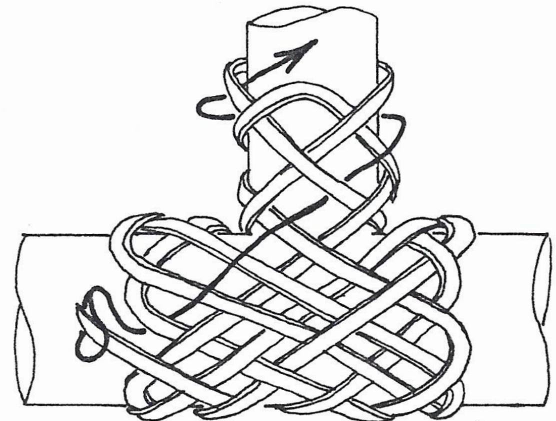


fig. 10

13. U O3 U O U

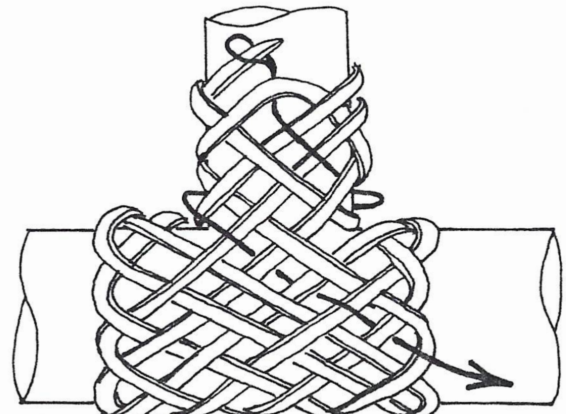


fig. 11

14. U O2 U O2 U2 O U O

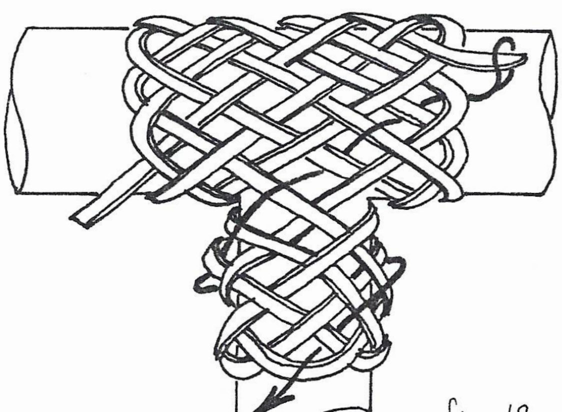


fig. 12

15. U2 O U O2 U O2 U O

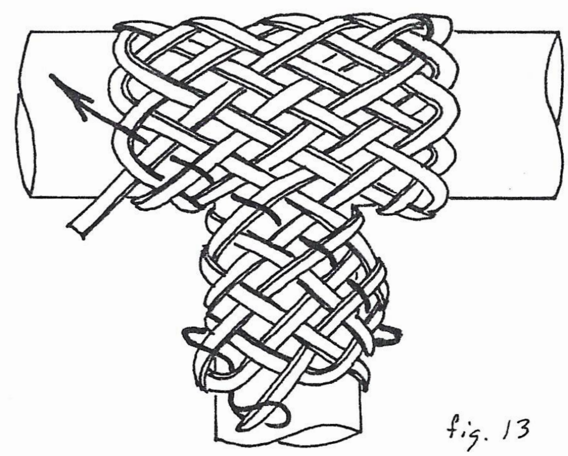


fig. 13

16. U O U O U O U O U O U O

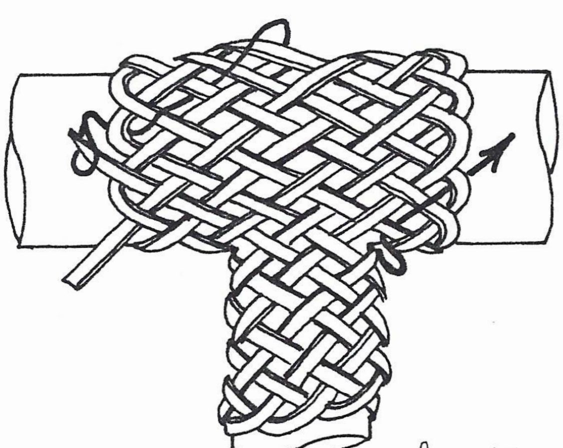


fig. 14

17. U O U O2 <u>U O</u> U O U
in the back

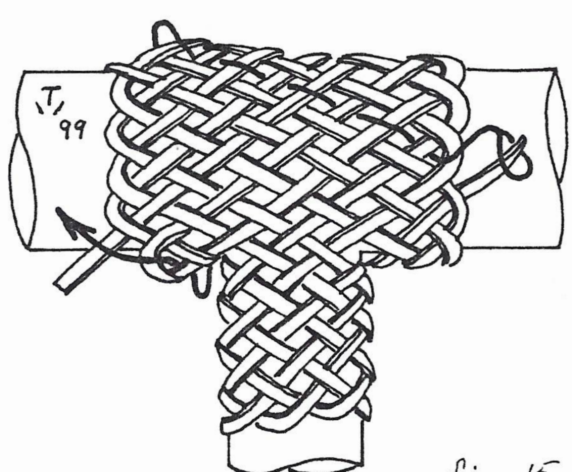


fig. 15

18. O U O U O U O U O U O

12 Part 7 Bight 1 pass Knot with a 6 Bight hole

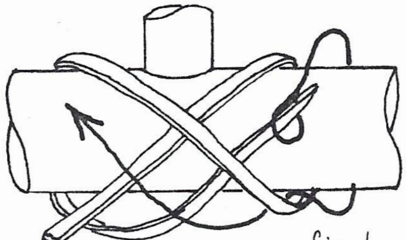


fig. 1

1. free run
2. O
3. U
4. U O2

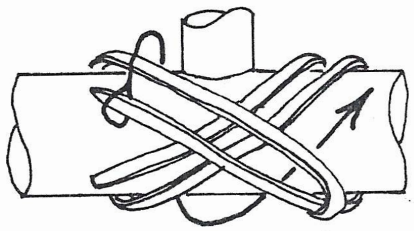


fig. 2

5. O U2

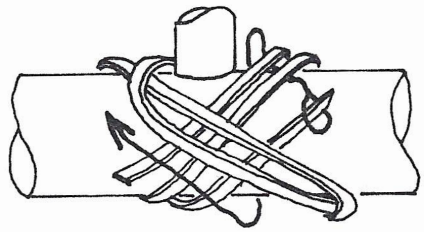


fig. 3

6. U2 O3

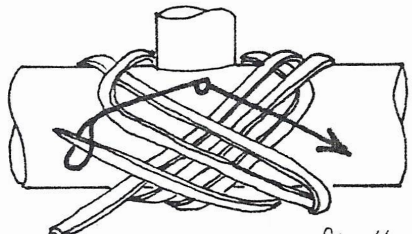


fig. 4

7. O2
8. O3

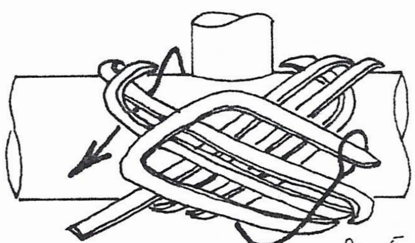


fig. 5

9. O3 U2

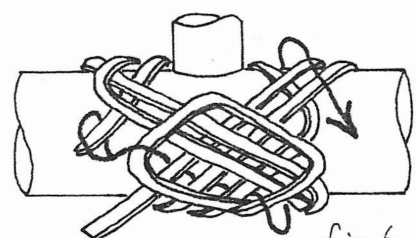


fig. 6

10. O U3 O3

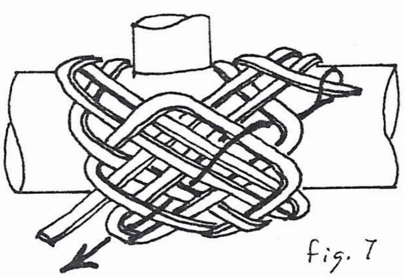


fig. 7

11. U O3 U O U

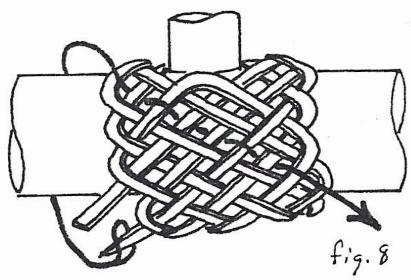


fig. 8

12. O U O U2 O U O2

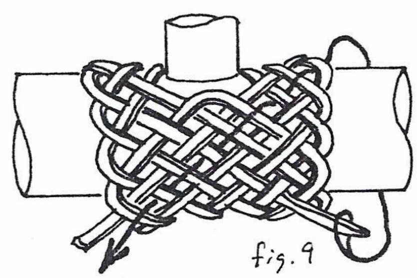


fig. 9

13. U O U O2 U O U O

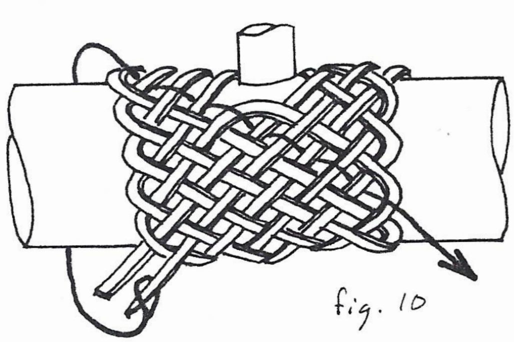


fig. 10

14. O U O U O U O U O U O

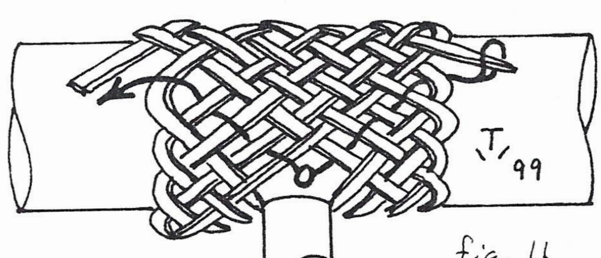


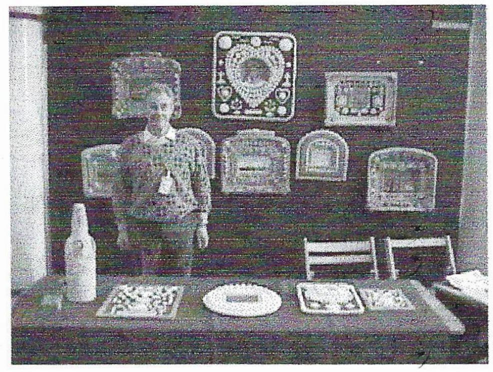
fig. 11

15. U O U O U
16. O U O U O

Joke of the Day

[Please accept this joke in the spirit in which it is sent]

One day a young cowboy and cowgirl decided to get married.
After the wedding they left for their honeymoon. While driving down the road, the new bride sees two cows having sex. The new bride asks, "What are they doing honey?"
The husband answers, "They're roping!"
She replies, "Oh, I see!" After a few more hours of driving they pass two horses having sex. Again the bride asks, "What are they doing honey?"
The husband answers, "They're roping!"
She replies, "Oh, I see!" Finally they arrive at their hotel. The couple washed up and started to get ready for bed. When they got in the bed, they started to explore each other's body. The bride discovers her husbands penis. "What is that?" she asks.
"That's my rope," he answers.
She slides her hands down further and gasps, "What are those?" she asks.
"They're my knots," he answers.
Finally the couple begins to make love. After several minutes the bride says, "Stop honey, wait a minute!"
Her husband asks, "What's the matter honey, am I hurting you?"
"No," the bride replies, "Undo those knots, I need more rope!"

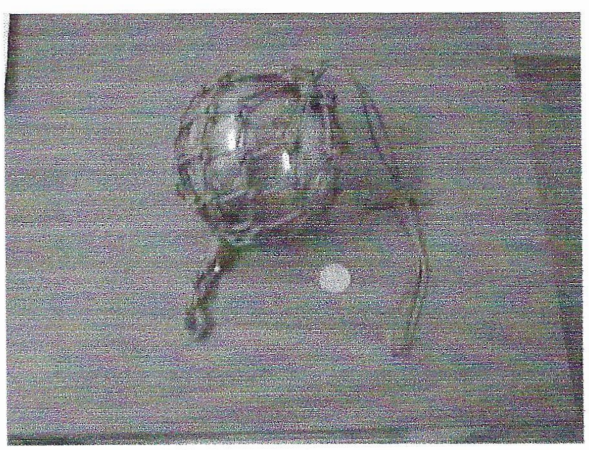


Mr. Bernard Catbush
Photo courtesy of Randy Penn

The work itself will not only serve many practical purposes but will also lead to the mastery of an artistic and interesting hobby. At the same time, the student will derive a deep sense of satisfaction from knowing how to make a knot properly and how to apply it to the correct situation.

Once a sound basic knowledge of the various ties has been acquired, the student will discover fascinating possibilities for creative development of advanced knot work. The success with which he masters the more advanced knot work depends to a great extent upon personal initiative and a genuine determination to learn the subject step by step."

Raoul Graumont
Handbook of Knots



This knot work is by Todd Thom.
Photos are courtesy of The Knot Shoppe

Knot Calendar

May 22 - National Maritime Day. Los Angeles Maritime Museum in San Pedro.

International Guild of Knot Tyers
Joseph Schmidbauer
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Corona, California 91720 USA